Complex Control Systems
ISSN 1310-8255 **Proceedings of the International Scientific Conference "Robotics & Mechatronics 2023"**

Modelling and simulation of direct-current motor for indoor wheeled mobile robot

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Abstract*—The thematic is in the engineering (mechatronics) branch, with application in the mobile robotics. In the article have been analysed: the mechanical energy of direct-current motor at motion, its corresponding mechanical and electrical parameters, in that way they will be used for a mechatronics system of a wheeled mobile robot. The goal criterion is the achievable dynamics parameters. This research can be used as a study of direct- current motor and simulative model in some lecture notes on mechatronics and automatics.

***Keywords*—Mechatronics system elements, Simulation and modelling, Simulation of PMDC motor, Torque and speed of electrical DC motor, Wheeled mobile robot.**

I. Introduction

In this report is studying the behaviour of a PMDC (Permanent Magnets Direct- Current) motor, embracing all its properties, both electrical and mechanical. In the article the motor is considered to be a pure linear mechatronic converter.

The topic is studying of PMDC motor as an immutable part of any indoor wheeled mobile robot, by means of a good simulation model.

The main goal is proposing two models, which being based on math relations, to simulate the almost linear demeanour of a PMDC motor and to be used to determine the mechatronic characteristics of this motor on the whole.

The question we pose is in what degree the proposed models are adequate to similar models from other studies.

The importance of this question is in the availability of mobile platform to develop some speed and the results obtained by simulation for a PMDC motor as driving device, to be applicable for its engineering designing and automatic control.

The related works: In [1] is given the Laplace transform for the linearised mathematical model of PMDC motor. The study ignores the presence of motor coil inductance because its negligible influence on the overall motor mechanical dynamics. While this assumption is true in general, the coil inductance has impact on the electrical controllability of the device by means of power drive electronics; So this is the main drawback of that model.

The state of the art: In [2] through [5] the impact of coil inductance is taken into account. It gives a bit more realistic model.

How will this study advance our knowledge? The proposed below model uses this inductance too: It will be necessary in the future, when to study the conjunction of PMDC motor with the H-bridge, (as its schematics resembles the letter H) when the control PWM (Pulse Width Modulation) changes.

The hypothesis we will test is: Are the proposed simulation models close on the similar models from other studies as well as on the analytical solution.

The manuscript consists of 4 parts: Theoretical model; Implementation; Experiments; Conclusion.

II. Methods and approaches

A. *Theoretical model of PMDC motor*

The physical model of PMDC motor is taken from [6], where it has been shown the model is linear. The equivalent circuit diagram in (Fig. 1) represents a PMDC motor. The used physical terms on the figure are explained below in the equations.

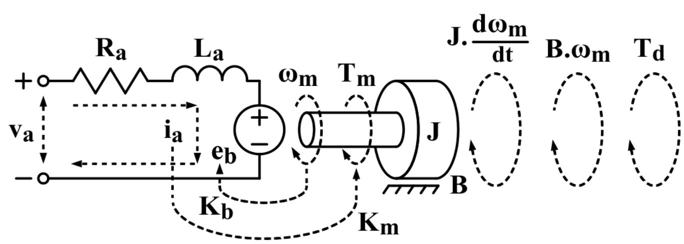

Fig. 1. Electrical and mechanical model of DC motor.

1) Equations of electrical balance

The armature is given as a circuit with resistance $R_a\ [\Omega]$ connected in series with an inductance $L_a\ [H]$; The electrical balance is defined as the armature voltage $v_a(t)[V]$ is equal to voltage drops caused by armature current $i_a(t)\ [A]$ flowing through armature resistance and inductance, plus a voltage source $e_b(t)\ [V]$, representing the back-EMF (Electro-Motive Force) in the armature, when the rotor rotates:

$$\mathrm{v}_a(t) = L_a \cdot \frac{di_a(t)}{dt} + R_a \,.\, i_a(t) + e_b(t) \quad (1)$$

If the angular displacement of motor shaft is denoted as $\theta_m(t)\,[rad]$: The back-emf is proportional to rotational speed of the rotor $\omega_m(t) = d\theta_m(t)/dt\,[rad/sec]$ by a coefficient $K_b\ [V/rad/sec]$, called back-EMF constant:

$$\mathrm{e}_b(t) = K_b \cdot \frac{d\theta_m(t)}{dt} = K_b \,.\, \omega_m(t) \quad (2)$$

According to the circuit diagram of (Fig. 1), the speed control of the DC motor is made at the armature terminals by means of the applied armature voltage:

$$\mathrm{v}_a(t) = L_a \cdot \frac{di_a(t)}{dt} + R_a \,.\, i_a(t) + e_b(t) \quad (3)$$

If the angular displacement of motor shaft is denoted as $\theta_m(t)\,[rad]$: The back-emf is proportional to rotational speed of the rotor $\omega_m(t) = d\theta_m(t)/dt\,[rad/sec]$ by a coefficient $K_b\ [V/rad/sec]$, called back-EMF constant:

$$\mathrm{e}_b(t) = K_b \cdot \frac{d\theta_m(t)}{dt} = K_b \,.\, \omega_m(t) \quad (4)$$

According to the circuit diagram of (Fig. 1), the speed control of the DC motor is made at the armature terminals by means of the applied armature voltage.

2) Equations of mechanical balance:

The developed motor torque must be in balance by the consumed mechanical torque, which include three terms:

- Torque caused by an inertial force (represented by the second moment of inertia $J\ [kg.m^2]$ for the load multiplied by the angular acceleration $d\omega_m(t)/dt$); Note this second moment includes the inertial moment J_m of the motor itself plus the inertial moment J_l of the load, i.e. $J = J_m + J_l$;
- Torque caused by a viscous friction (represented the viscous friction coefficient $B\ [N.m.sec/rad]$ multiplied by the angular speed $\omega_m(t)$); Same note applies for the friction, it includes friction coefficient B_m of the motor plus the friction coefficient B_l of the load, i.e. $B = B_m + B_L$;
- Some torque $T_d(s)\ [N.m]$, caused by mechanical disturbances (This torque is artificially introduced as a disturbance variable for some analyses of dynamics for the mechatronics tandem of motor and load, for the purpose of control system research).

Thus the developed motor torque is given as:

$$\mathrm{T}_m(t) = J \cdot \frac{d\omega_m(t)}{dt} \ +\ B \,.\, \omega_m(t) \ +\ T_d(t) \quad (5)$$

3) United model representation

The dependent variables of the system will be $i_a(t)$, and $\omega_m(t)$. By substituting and eliminating all other variables from Eq. (1) through Eq. (5), a system of two differential equations is obtained:

$$\frac{di_a(t)}{dt} = -\frac{R_a}{L_a} \cdot i_a(t) - \frac{K_b}{L_a} \cdot \omega_m(t) + \frac{1}{L_a} \cdot v_a(t)$$

$$\frac{d\omega_m(t)}{dt} = \frac{K_m}{J} \cdot i_a(t) - \frac{B}{J} \cdot \omega_m(t) \frac{1}{J} \cdot T_d(t) \quad (6)$$

Note that, the independent variables (sometime called hyper-parameters) are $v_a(t)$ and $T_d(t)$: In following analyses, there will be applied the control input and the disturbance respectively.

4) Equations in Laplace domain

Now we will cite an analytical solution that will be useful when to compare with simulated models of the PMDC motor.

In [7] is stated the analytical link between the output (rotational speed) and the two hyper-parameters (input for controlling armature voltage as well as torque disturbances) is given as:

$$\Omega(s) = \frac{\frac{K_m}{R_a.J} \cdot V_a(s)}{\left(\frac{L_a}{R_a}\right) \cdot s^2 + \left(1 + \frac{B.L_a}{R_a.J}\right) \cdot s + \frac{K_m.K_b + R_a.B}{R_a.J}} - \frac{\left\{\left(\frac{L_a}{R_a}\right) \cdot s + 1\right\} \cdot \frac{1}{J} \cdot T_D(s)}{\left(\frac{L_a}{R_a}\right) \cdot s^2 + \left(1 + \frac{B.L_a}{R_a.J}\right) \cdot s + \frac{K_m.K_b + R_a.B}{R_a.J}} \quad (7)$$

5) Motor behaviour in time

According [7], in order to determine the speed response $\omega(t)$, due to the individual inputs, a superposition has been applied in 2 steps.

- Step 1

For $T_D = 0$ (i.e. at lack of disturbance), we get:

$$\Omega(s) = \frac{K_m.V_a(s)}{L_a.J.s^2 + (R_a.J + B.L_a).s + (K_m.K_b + R_a.B)} \quad \{ T_L(s) = 0 \} \quad (8)$$

Now for an applied voltage $V_a(t) = V_{max}$, such that $V_a(s) = V_{max}/s$ (i.e. as a step function of amplitude V_{max}, where $V_{max} = const$) and using the initial and final value theorems

$$\lim_{t\to 0} \omega(t) = \lim_{s\to\infty} s.\Omega(s) = 0$$
$$\lim_{t\to\infty} \omega(t) = \lim_{s\to 0} s.\Omega(s) = \frac{K_m.V_{max}}{K_m.K_b + R_a.B} \quad (9)$$

These initial and final values will be used when we compare with the simulated models of motor.

- Step 2

Suppose the armature voltage $V_a(t) = V_{max}$ has been attached for long time, i.e. the motor has well settled speed. Then we apply a disturbing torque $T_D(s)$ to the rotor, i.e. to the mechanical part. In that case the speed response changes to:

$$\Omega(s) = \frac{K_m.V_{max}}{K_m.K_b + R_a.B} - \frac{(L_a.s + R_a).T_D(s)}{L_a.J.s^2 + (R_a.J + B.L_a).s + (K_m.K_b + R_a.B)} \quad (10)$$

If the disturbing torque is a step function (i.e. $T_D(s) = T_{max}/s$, $T_{max} = const$), then the initial and final value theorems lead to

$$\lim_{t\to 0} \omega(t) = \lim_{s\to\infty} s.\Omega(s) = \frac{K_m.V_{max}}{K_m.K_b + R_a.B}$$
$$\lim_{t\to\infty} \omega(t) = \lim_{s\to 0} s.\Omega(s) = \frac{K_m.V_{max}}{K_m.K_b + R_a.B} - \frac{R_a.T_{max}}{K_m.K_b + R_a.B} \quad (11)$$

A practical note: In order the motor to run (i.e. $\omega(t) \geq 0$): The value of $T_D = T_{max}$ should never exceed the motor stall torque (otherwise the motor will become generator, because of the disturbing torque). This physical condition imposes the speed obtained by the final value theorem (the last equation) never to become negative. It is represent as: $K_m \,.\, V_{max} - R_a \,.\, T_{max} \geq 0$, which sets limits on the magnitude of the torque:

$$0 \leq T_{max} \leq \frac{K_m}{R_a} \cdot V_{max} \quad (12)$$

The obtained final values and limitation are important when we compare with the simulated models of motor.

B. Implementation

1) Major design decisions

In order to simulate PMDC motor, two simulation models has been constructed: First one is based on Laplace transformed equations; The other model is using differentiating operation and feed-backs.

- Simulation model 1

In [7] has been shown, after applying Laplace transform, the relationships between the hyper-parameters images $\Omega(s)$, $V_a(s)$, and $T_D(s)$ (of the originals $\omega(t)$, $v_a(t)$, and $T_d(t)$ respectively) can be given as a block diagram (Fig. 2). For brevity, we will call it 'Laplace model' in the following text.

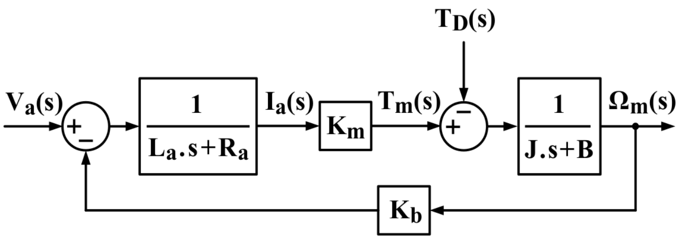

Fig. 2. Model block diagram of a PMDC motor in Laplace domain.

- Simulation model 2

In the most simulators, based on solvers over the integral representation, it is preferable to use the differential form of equations, not the Laplace one, as the former gives much quicker solution and greater simulation speed. For that case a equivalent block diagram has been constructed, using several feed-backs and differentiating operations (Fig. 3). For brevity, this simulation model we will name 'Electrical model' in the following lines of article.

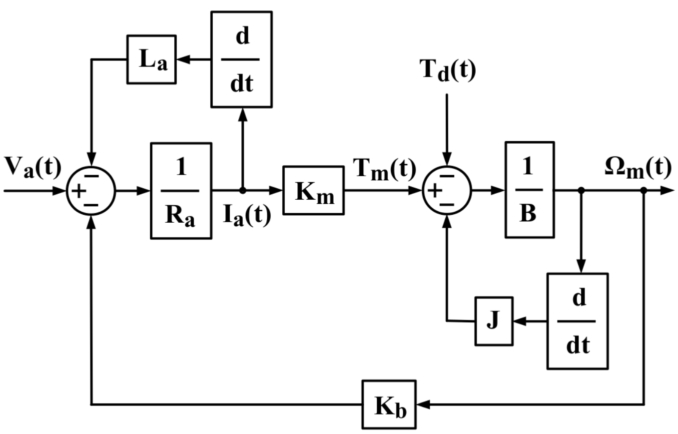

Fig. 3. Model block diagram of a PMDC motor based on feed-backs and derivatives.

2) *Models into software simulator*

LTspice® (Analog Devices Inc.) has been used for the purposes of simulation, a short introduction of it is given in [8]. It is an electronic circuits simulator, with SPICE language definitions. We use it because it can work with derivatives and Laplace domain, combined with fully featured and precise simulation of power control electronics (H- bridge).

The created models are of type 'subcircuit', i.e. resembling building 'brick' or computer language 'subroutine' and will be used for future experiments.

Subcircuit of PMDC motor, in Laplace domain is given in (Fig. 4). Inputs are '*Va*', '*Td*' for armature voltage and disturbing torque respectively; Outputs are '*Ia*', '*Tm*', '*Om*' accordingly for armature current, developed motor torque and motor rotational speed

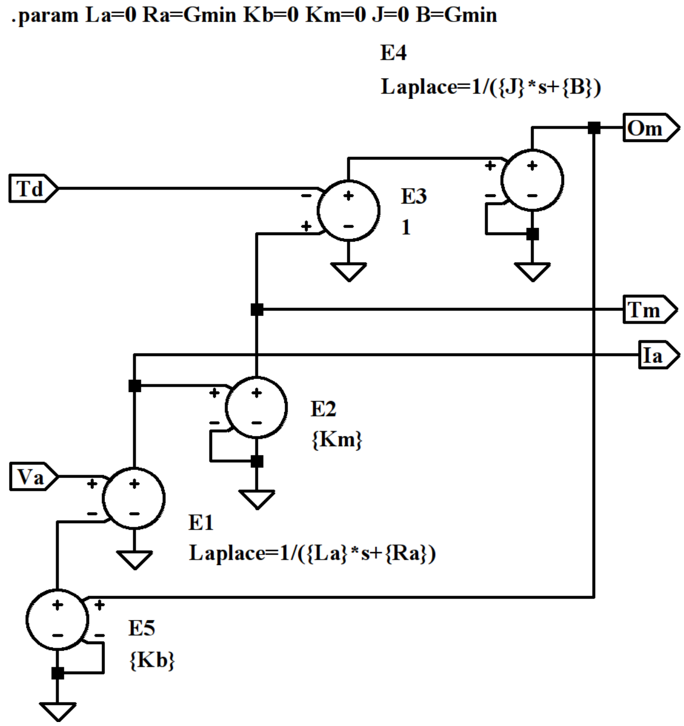

Fig. 4. Simulation model subcircuit of PMDC motor in Laplace domain.

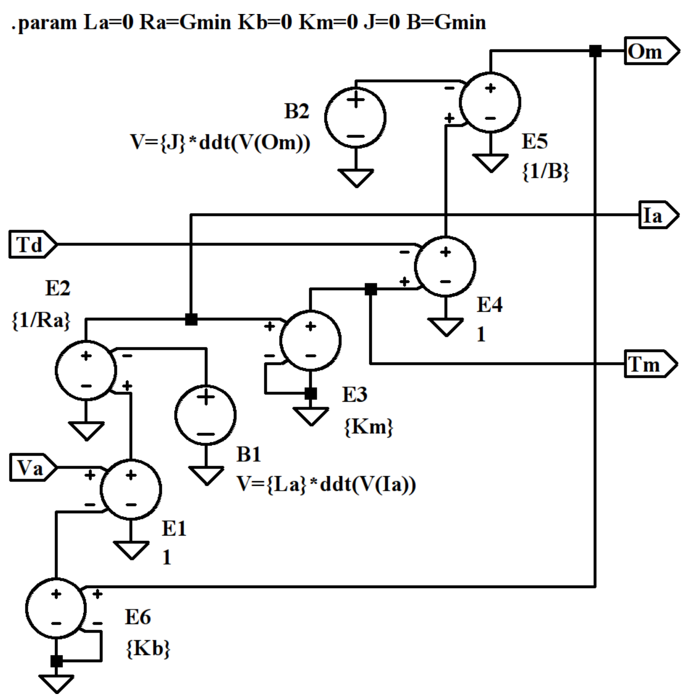

Fig. 5. Simulation model subcircuit of PMDC motor, based on feed-backs and derivatives.

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ISSN 1310-8255 Proceedings of the International Scientific Conference "Robotics & Mechatronics 2023"

The math operations have been done by voltage sources (noted as '*En*'), having defined gain or making Laplace transformation of its input signal; The voltage outputs have 'encircling' symbol notation, while signs '+' and '-' represent voltage polarity of input or output. The '*.param*' directive defines the parameters passed to the subcircuit and their default values if they have been omitted. The brackets '*{}*' around a phrase impose the phrase numerical value to be calculated before simulation.

Subcircuit of PMDC motor, in Laplace domain is given in (Fig. 5). The difference between the two subcircuits is in their structure as well as the use of so called 'behavioural' voltage sources: They have been noted as 'Bn' and work directly by a formula over the node voltages.

3) *Explanation of experiments*

The goal is to compare the analytical solution given in (Eq. 7) with the results obtained by means of experiments with the simulated model, as well as with experiments done by other autors. In [9] are given experiments on a PMDC motor having data:

- $J = 0.02$;
- $B = 0.2$;
- $K_b = 0.02$;
- $K_m = 0.02$;
- $R_a = 2$;
- $L_a = 0.4$;

The calculated analytical solution in [9] is:

$$\frac{\dot{\theta}(s)}{V(s)} = \frac{0.02}{0.008\ s^2 + 0.12\ s + 0.4004} \qquad (10)$$

The same math we obtain from (Eq. 7), at lack of disturbing torque, i.e. $T_D(s) = 0$. Additionally, eq. (8) and (9) give us analytical expectations about models response for input step signal on V_a input.

4) *Experimental 'jig'*

In order to compare the analytical solution in eq. (10) with the two models a schematic ('jig') that facilitates simulation have been done, as given in (Fig. 6): Subcircuits *X1* and *X2* are the two models with given respective parameters; Voltage source *E1* is the analytical solution in Laplace domain; Impulse voltage source *V1* creates a unit step of voltage (*0 V* to *1 V*, fronts of *1 pSec*, duration of

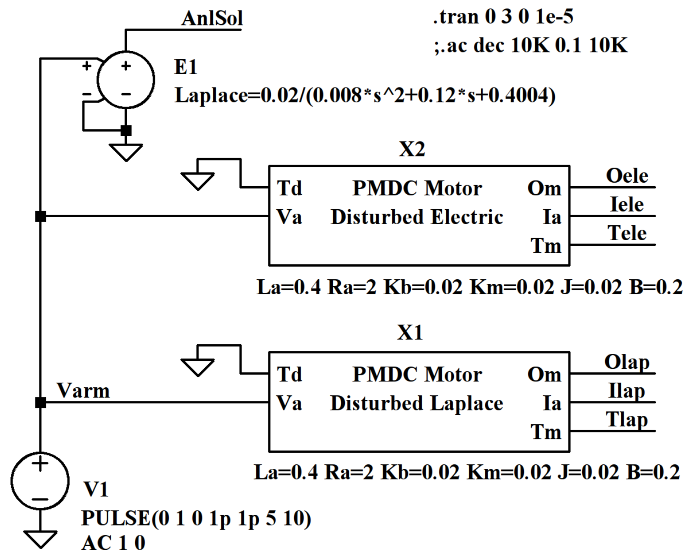

Fig.6. Schematic "jig' facilitating simulation for use in both time and frequency domains.

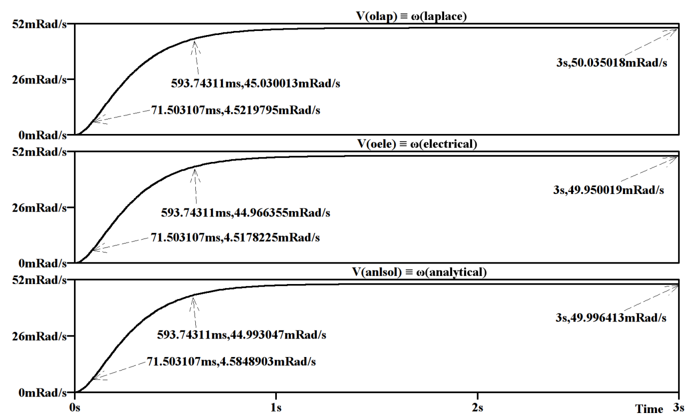

Fig.7. Response of rotational speed to unit step in armature voltage for the analytical solution and the two simulation models.

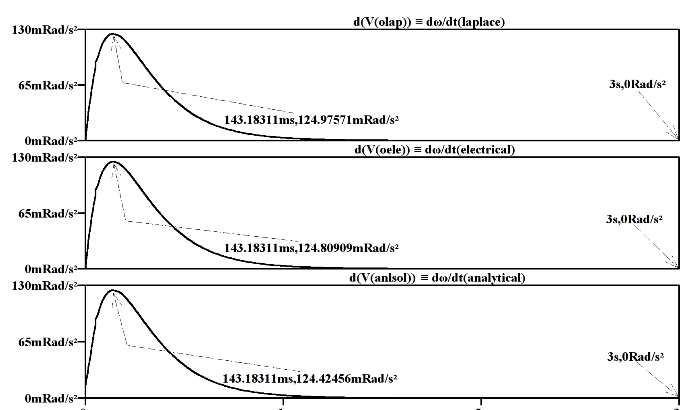

Fig. 8. Rate of change for the rotational speed because of unit step change of armature voltage, for the analytical form and both models.

states *5 Sec*, repetition period *10 Sec*); Directive '*.tran*' ensures *3 Sec* of transient simulation, time step is 10^{-5} *Sec;* Directive '*.ac*' is intended for future experiments in frequency domain, but it is not activated (it is started with semicolon, meaning just comment and not executed); Output '*AnlSol*' is the analytical solution; Outputs noted by '*O*' give the rotational speed, these with '*I*' – the armature current, with '*T*' – the torque developed by motor; The last letters ('*lap*' and '*ele*') denote which signal to which model belongs (Laplace or Electrical).

5) *Experimental results*

The responses of the analytical solution and the motor models to unit step on the armature voltage are given in (Fig. 7). On abscissa is time, ordinates represent the rotational speed of motor. Over each graph the presented value of simulated output and its physical meaning is given. On each graph are noted the final steady state value (in right) as well as the moments when the speed reaches 10% and 90% of its steady state value (these two points given in left). The three characteristic points are necessary for the object identification and parameters calculation for its future control algorithms (e.g. PI, PID, etc.).

In (Fig. 8) is given the rate of change (the derivative) for the motor rotational speed at the same input impact. On ordinates is placed the dimension of this rate of change. On each graph are noted the final state derivative (in right) as well as the maximum of the derivative (in left). These two points are necessary for identification purpose too.

6) *Experiments interpretation*

The characteristic points for the analytical solution and the two models show some little differences in their values. These discrepancies are after the 3-th digit of the results, i.e. we have some satisfactory coincidence, sufficient for engineering purposes.

The fact of such errors lies beneath the simulation solver tuning. Its internal variables, defining the computational accuracy, such as: Relative Error, Dynamic Range Error, Fourier Analyses Window, Fourier Steps Number, Maximal Time Step, etc. have impact on solver punctuality, as explained in [8]. In the experiments LTspice has been used with default values of these parameters, i.e. with standard tune for engineering simulations.

The behavior of the speed derivatives shows non-negative values with only one peak. It means that the speed behave with no overshoot nor oscillation. This is typical for PMDC motor, which is defined in theory as over-damped system of second order. This experimental fact confirms again the correctness of both the simulation models.

III. CONCLUSION

The proposed two models are good enough for simulation purposes intended for some applied scientific research (engineering). They can be used as a benchmark for some comparative analyses with other models for PMDC motors.

REFERENCES

[1] Siciliano B., Khatib O.; Springer Handbook of Robotics, 2nd edition, pp.78-80

[2] El Tohamy I.A., Ahmed A., El Koshairy D.; Enhancement of the Dynamic Performance of a DC Motor using Fuzzy Logic Algorithm; International Journal of Engineering Research & Technology (IJERT), Vol. 8 Issue 10, October-2019; ISSN: 2278-0181

[3] Miková Ľ., Virgala I., Kelemen M.; Speed Control of DC Motor ; American Journal of Mechanical Engineering, 2016, Vol. 4, No. 7, pp. 380-384 ; Science and Education Publishing ; DOI:10.12691/ajme-4-7-27

[4] Dhinakaran P., Manamalli D.; Novel strategies in the Model-based Optimization and Control of Permanent Magnet DC motors; Journal of Computers and Electrical Engineering 44 (2015) pp. 34–41; ScienceDirect , Computers and Electrical Engineering; Elsevier;

[5] Yildiz A.B.; Electrical equivalent circuit based modelling and analysis of direct current motors; Journal of Electrical Power and Energy Systems 43 (2012), pp. 1043–1047 ; SciVerse ScienceDirect , Electrical Power and Energy Systems; Elsevier;

[6] Golnaraghi F., Kuo B.C.; Automatic Control Systems, 9-th edition; John Wiley & Sons, Inc.; ISBN-13 978-0470-04896-2; pp.201-205

[7] Golnaraghi F., Kuo B.C.; Automatic Control Systems, 9-th edition; John Wiley & Sons, Inc.; ISBN-13 978-0470-04896-2; pp.289-292

[8] Alonso G.; Get up and running with LTspice; Analog Dialogue Technical Journal; dec 2019 , vol 53; Analog Devices Inc.

[9] Shamshiri R.R.; A Lecture Note on DC Motor Speed Control; EML 4312: Control of Mechanical Engineering Systems; Fall 2009, University of Florida