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Investigation of the stability of a wheeled mobile robot with a differential drive following a set trajectory

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***Abstract*— The object of the study is a wheeled mobile robot with differential drive wheels and castor support wheel. The article examines the problems of stability when the robot moves along a set trajectory. A simulation based on a kinematic and dynamic mathematical model was made, in which the longitudinal and transverse stability were calculated. Limit of longitudinal and transverse acceleration for movement along a set trajectory are calculated. The simulation shows that the maximum accelerations do not exceed the limit values guaranteeing the stability of the robot during the movement along the set trajectory.**

***Keywords*— Stability, wheeled mobile robot.**

I. Introduction

Wheeled mobile robots, due to their multipurpose, perform a wide range of tasks. One subgroup of them is the robots with differential wheel drive. They have good maneuverability, which is an advantage when working in narrow spaces, with many obstacles. Their construction is simplified; this facilitates their operation and extends the periods between necessary repairs and servicing. However, there are some challenges related to their stearing and achieving the necessary stability in motion. In this article, we consider some of these problems.

The article is organized as follows: in the second part, we have placed an implementation and description of part of the mathematical model of the robot, as well as its location in the space. In the third part, through numerical experiments, we investigate the accelerations to which the robot is subjected and the smoothness of the trajectory described by it, while it is in the mode of following a set trajectory. In the last part, we draw conclusions about the performance of the model.

II. Methods and materials

In Fig. 1.2 presents a robot with a differential drive on the two rear wheels and a front castor-type wheel, which moves along a set trajectory S in the plane $O_g x_g y_g$ of the global coordinate system $O_g x_g y_g x_g$. Point C (Fig. 2.2) represents the geometric center of the rear axle. We choose it to follow the trajectory S. Let point A be the support of a front wheel (we assume that the contact of the wheels with the road is a point, not a spot), points L and R are the supports of the left and right wheels, respectively. L and R follow trajectories S_L and S_R, that are equidistant to S.

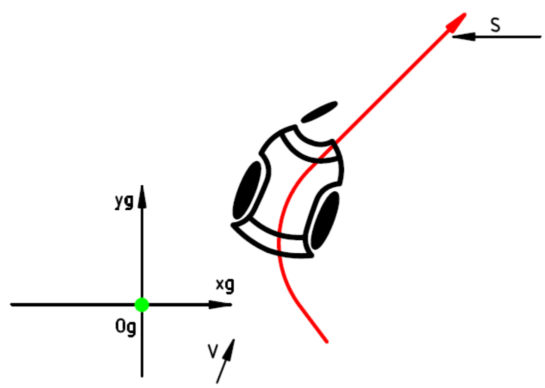

Fig. 1.2 Position of the robot and the set trajectory in the horizontal plane of movement

The Cxyz coordinate system is coupled to the robot body. The x axis is contained in the longitudinal plane of symmetry of the robot coinciding with the Cxz plane. We assume that the mass center m_c lies on the x-axis. The axis vm_c is perpendicular to the segment $\overline{m_c O_{turn}}$, which is the moment radius of turn of the center of mass. This axis determines the direction of the velocity of the center of mass. We determine the angle $\theta_{V_{m_c}}$. Then:

$$V_{m_c} = \frac{V}{\cos\theta_{V_{m_c}}} \tag{1}$$

We determine the projections of V_{m_c} on the x and y axes of the coupled coordinate system:

$$V_{m_c}x = V_{m_c}\cos\theta_{V_{m_c}};\ V_{m_c}y = \sin\theta_{V_{m_c}}$$

At $\overline{CO_{turn}} = const$, i.e. the robot moves in a circle centered at O_{turn}, only centrifugal acceleration and centrifugal force are present. For the center of mass, at a given peripheral velocity, we have a centrifugal force which is:

$$P_c = mr\left(\omega_{O_{turn}}\right)^2, \tag{2}$$

where m is the mass of the robot centered at m_c,
$r = \overline{CO_{turn}}$ and

$$\omega_{O_{turn}} = \frac{V_{m_c}}{\overline{CO_{turn}}}. \tag{3}$$

Given that the centrifugal acceleration along the axes of the coupled coordinate system is:

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$$a_{m_c}x = r\left(\omega_{O_{turn}}\right)^2 \sin\theta_{V_{m_c}}\,;\; a_{m_c}y = r\left(\omega_{O_{turn}}\right)^2 \cos\theta_{V_c} \quad (4)$$

respectively:

$$P_c x = P_c \sin\theta_{V_{m_c}};\, P_c y = P_c \cos\theta_{V\,m_c}. \quad (5)$$

Thus, in the special case when $\overline{CO_{turn}} = 0$ and $\overline{Cm_c} \neq 0$, the centrifugal force will not act along the transverse axis, but only along the longitudinal axis x of the coupled coordinate system; respectively at $\overline{CO_{turn}} \neq 0$; $\overline{Cm_c} = 0$, the centrifugal force will act only along the y-axis. If the robot makes a turn at $\overline{CO_{turn}} = 0$; $\overline{Cm_c} = 0$, then no centrifugal force will act on the center of mass. Of course, the mass moment of inertia I must be taken into account.

If $\overline{CO_{turn}} \neq const$, then $V_{m_c}x \neq const, V_{m_c}y \neq const$, respectively $\dot{V}_{m_c}x \neq 0; \dot{V}_{m_c}y \neq 0$, then for the longitudinal and transverse acceleration we have:

$$a_{lon} = \dot{V}_{m_c}x + a_{m_c}x;\; a_{tran} = \dot{V}_{m_c}y + a_{m_c}y. \quad (6)$$

Limit values for longitudinal and transverse acceleration are determined according to the methods described in [4].

The axes of rotation of the left and right wheels coincide with the y-axis. Point O_{turn} lies on the y-axis; it is the instantaneous center of the arc of the set trajectory. The definition area of a point O_{turn} matches the set of points belonging to the y-axis. In the particular case of rectilinear motion O_{turn} coincides with infinity. With this construction scheme, the special case of a turn with zero radius is also possible, i.e. $C \equiv O_{turn}$. Actually, the equidistance of the trajectories is lost when $O_{turn} \in \overline{LR}$, and $\overrightarrow{V_L}$ и $\overrightarrow{V_R}$ become in opposite directions.

We introduce a cornering response coefficient with which the cornering response of various structures built according to the considered kinematic scheme can be compared:

$$k_R = f(\overline{CO_{turn}}; \overline{LR}; \overline{Cm_c}; \overline{CA}). \quad (7)$$

The purpose of this coefficient is to facilitate the selection of the geometrical parameters of the structure; the mass moment of inertia I and the rolling resistance of the wheels are accounted for independently of k_R.

More specifically, the coefficient can be written:

$$k_R = \frac{\overline{CO_{turn}}}{\overline{LR} + \overline{AC} + \overline{m_c C}}. \quad (8)$$

If it is necessary to correct the influence of some components on the right side of the equation, they can to be multiplied by an appropriate weighting coefficient.

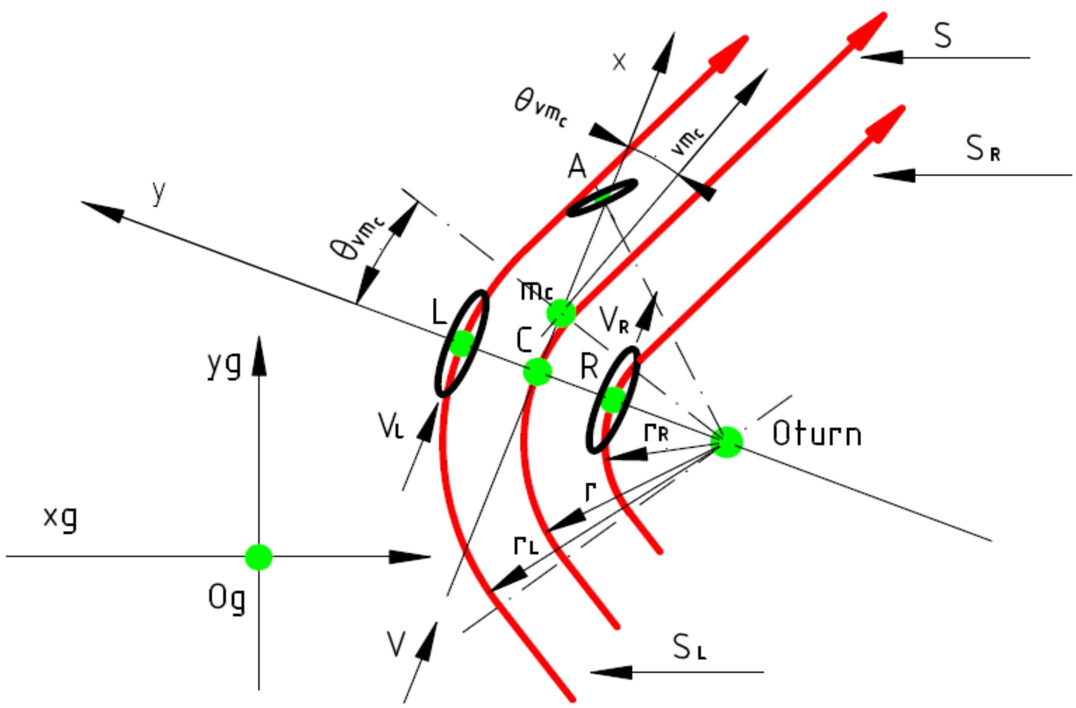

Fig. 2.2 The robot and the three trajectories

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III. EXPERIMENTS AND RESULTS

We calculate the necessary torques so that the mobile robot moves along the reference trajectory:

xr = 1,1 + 0,7 sin (2π / 30);

yr = 0,9 + 0,7 sin (4π / 30).

The simulation calculates the torques of the robot using the inverse dynamic model and plots the motion trajectory in Fig. 1.3.

The parameters of the robot are mass m = 0,25 kg, mass moment of inertia I = 0,01 kgm2, length L = 0,10 m, wheel radius r = 0,015 m and wheel track L = 0,04 m.

Figures 2.3 to 7.3 show the simulation results. With the parameters of the robot selected in this way, the transverse and longitudinal stability of the robot are ensured.

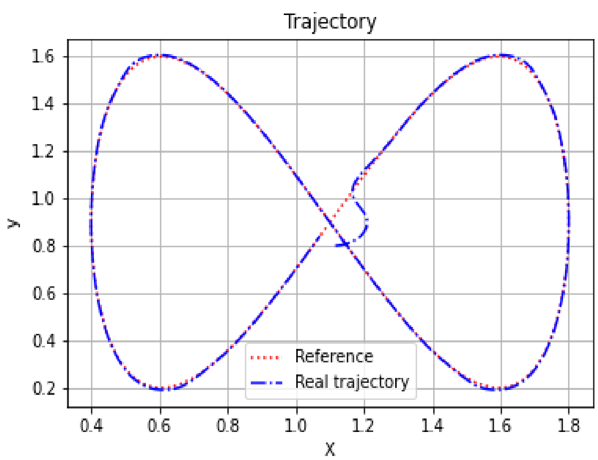

Fig. 1.3

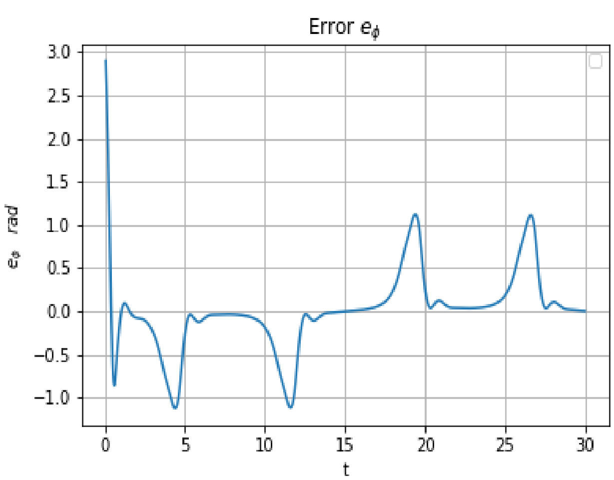

Fig. 2.3

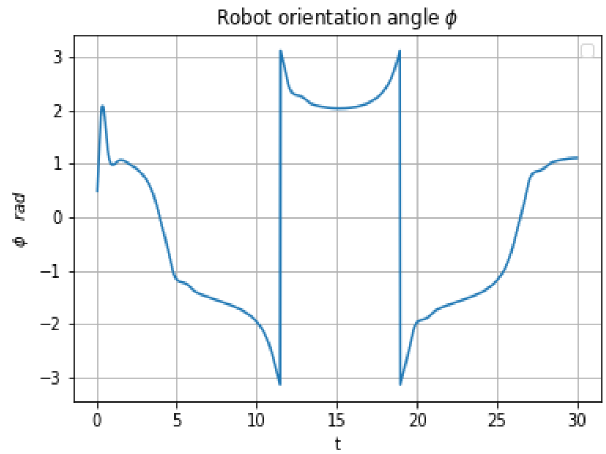

Fig. 3.3

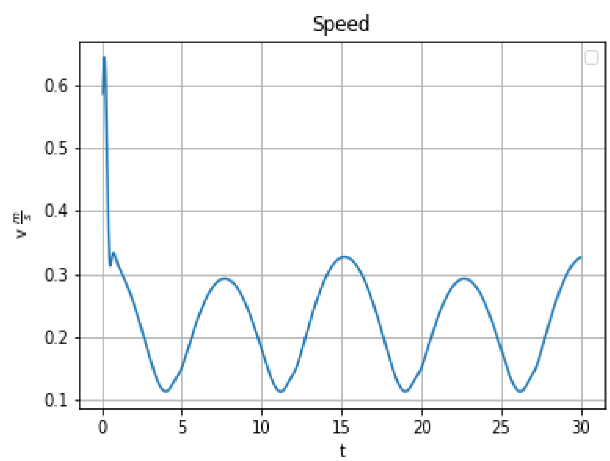

Fig. 4.3

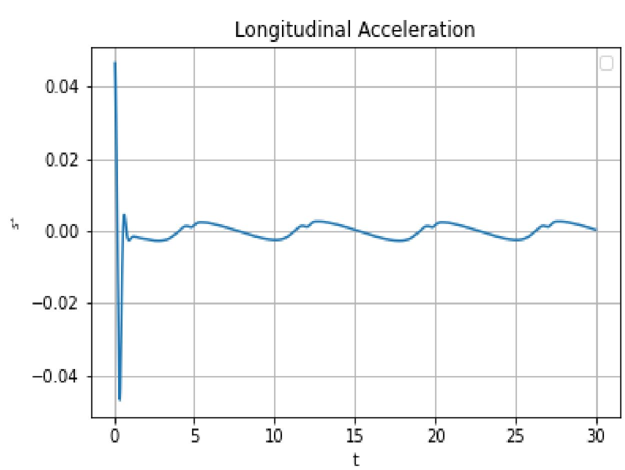

Fig. 5.3

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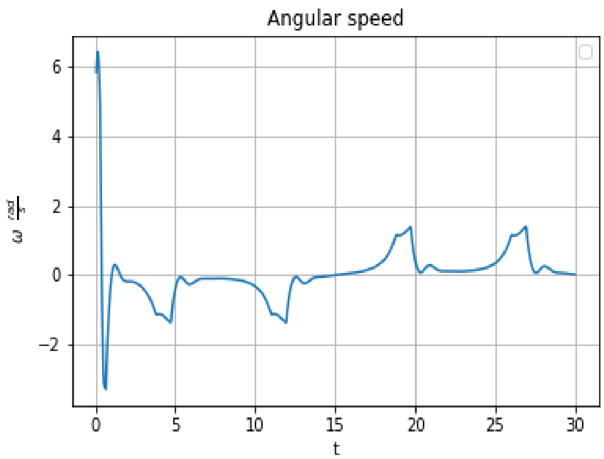

Fig. 6.3

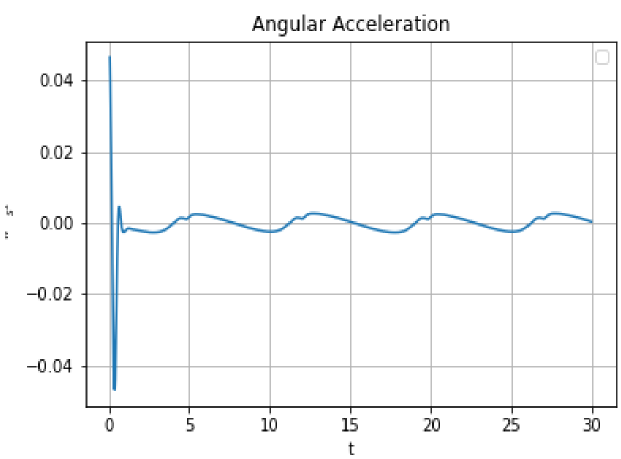

Fig. 7.3

IV. Conclusion

The problems of stable control of a robot with a differential drive on both wheels and a support, castor-type wheel are investigated. A model of this type of robot was built based on the principles of kinetostatics. The longitudinal and transverse stability of wheeled mobile robots, depending on their geometric proportions, as well as depending on the forces acting on them, have been added to the model.

The influence of the geometrical parameters of the robot on its longitudinal and transverse stability was investigated. Through numerical simulation, it is shown that the modeled robot has parameters where the longitudinal and transverse stability are ensured.

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